Notation used in Principia Mathematica Proof Checking Program

by Dennis Darland

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The program is not an automated theorem prover, rather it verifies that abbreviated proofs can be expanded into full proofs of the theorems. So far, it has only been applied through *5.75. (Not all this is re-entered yet, and there are a couple difficulties).

The program was written by Daniel J. O'Leary by 1982. It was re-entered by myself, Dennis J. Darland in 2017. There have been typos corrected and minor changes to the I/O, but no intentional changes made to O'Leary's logic made to this point (7/7/2017).

O'Leary uses Jan Lukasiewicz's polish notation, as it is much more suitable for computer manipulation.

There are 2 primitive ideas.

Apq   for p or q
and
Np   for not p

Also defined are

Cpq   for p materially implies q
Kpq   for p and q
and
Epq   for p is materially equivalent to q.

p,q, r and s are used as elementary propositions

Definitions are preceded by a line containing "DEF"
e.g.

DEF
*1.01 Cpq=ANpq

There are also axioms (called primitive propositions in PM) which are preceded by "AX"
e.g.

AX
*1.2 CAppp Taut

The 1st proof abbreviation is:

2&*1.2 p/Np+(1)
(1)+*1.01 q/Np;*2.01
*2.01 CCpNpNp Abs

Here the 2& merely indicates to the program that there are 2 lines in the proof abbreviation.
(If there is only one line this is omitted.)
After those two lines is the thesis whose proof is being checked.
*1.2 refers to the previous axiom.
CAppp
p/Np indicates to substitute Np for p in that axiom.
Doing that yields.
CANpNpNp
+(1) indicates to save that result as (1)
In the next line, we apply the definition *1.01 to this (1).
*1.01 was Cpq=ANpq
So by the definition:
(substituting Np for q)
ANpNp = CpNp (reversing order)
substituting CpNp in for ANpNp in (1) gives
CCpNpNp
which is *2.01

The next notworthy use of notation I notice is in *2.06
3&*2.04 p/Cqr, q/Cpq, r/Cpr+(1)
*2.05+(2)
(1)+C(2)-*2.06
*2.06 CCpqCCqrCpr Syll

This results in the proof:
PROOF
        1 CCpCqrCqCpr   *2.04   (Comm)
            p/Cqr, q/Cpq, r/Cpr
   (1) 2 CCCqrCCpqCprCCpqCCqrCpr   SUB
   (2) 3 CCqrCCpqCpr   *2.05   (Syll)
        4 CCpqCCqrCpr   MP: 2, 3

The additional notation was in the last line of the proof "(1)+C(2)-*2.06"
One has gotten (1) from *2.04 by making the substitutions indicated in the 1st line.
Then *2.05 becomes (2).
and (2) is the antecedent in (1) giving the conclusion 4 or *2.06

In this case X+CY-Z
indicates you have X and Y and Y is CXZ so you can conclude Z
X, Y, Z can be in the form of (n) or *n.m